Fast iterative contourlet thresholding for compressed sensing MRI

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This paper proposes to use contourlet as a sparse transform and combine it with fast iterative shrinkage/threshold algorithm (FISTA) for compressed sensing magnetic resonance imaging (CS-MRI) reconstruction. The proposed method not only inherits the simplicity and effectiveness of the original FISTA but also owns the sparse curve representation ability of contourlet. Simulation results validate the superior performance of the proposed method in terms of reconstruction accuracy and computation time.

Introduction: Computation efficiency and image accuracy are two fundamental goals in the image processing in general and in the reconstruction of compressed sensing MRI specifically. The fast iterative shrinkage/threshold algorithm (FISTA) is widely used in recent years for linear inverse problems including image denoising [1,2], image deblurring [1,2], compressed sensing (CS) MRI reconstruction [3,4] and CS remote sensing imaging [5]. The FISTA is an efficient algorithm has a faster convergence speed than some traditional methods such as iterative shrinkage/threshold algorithm (ISTA) and two-step ISTA (TWISTA) [6]. Meanwhile, the FISTA is able to obtain better results in terms of accuracies than these methods when applied to simple regularization problems such as image denoising and deblurring [1-2] than traditional iterative threshold algorithms [1-2]. When FISTA and composite splitting techniques [3] are combined, they can be used to solve the composite total variance (TV) and wavelet sparsity regularization problems such as the reconstruction of compressed sensing MRI [3]. Nevertheless, TV may result in loss of texture [7], and wavelet may fail to represent image curves though it can enforce point singularities and isotropic features of images [4-5]. Curvelet, on the other hand, outperforms wavelet in representing curve-like features of images and was used to replace wavelet sparse transformation in the native FISTA to improve the effectiveness of remote sensing imaging reconstruction [5]. A clear drawback of curvelet sparse transform is the long computation time due to its high redundancy. Contourlet, another image geometric transform akin to curvelet, has much less redundancy but with efficient sparse representation of curves and has been, therefore, utilized for CS-MRI in [4]. Although the use of contourlet in [4] is simple and effective, the iterative threshold reconstruction algorithm in [4] is slow. This letter proposes to use contourlet as sparse transform of FISTA for CS-MRI, aiming to improve the quality of reconstructed image and computation efficiency.

Theory: The CS-MRI reconstruction problem in this letter can be formulated as

$$\hat{x} = \arg\min_{x} \left\{ \frac{1}{2}\|Rx-b\|^2 + \alpha\|\Phi x\|_1 \right\} \tag{1}$$

where x is the underlying MR image to be reconstructed, R is the partial Fourier transform, b is the undersampled measurement of k-space data, Φ is the contourlet transform, and α is a trade-off parameter to tune data fidelity term and sparsity regularization term in (1).

The proposed fast iterative contourlet thresholding algorithm (dubbed as FICOTA) is outlined in algorithm 1 and is utilized to solve (1).

Algorithm 1. FICOTA

Input: $\rho=1/L_f$, α, $t^1=1$, $r^1 = x^0$
for k=1 to K do

$x_g = r^k - \rho\nabla f(r^k)$

$x^k = prox_\rho(2\alpha\|\Phi x\|_1)(x_g)$

$x^k = project(x^k,[l,u])$

$t^{k+1} = \left(1+\sqrt{1+4(t^k)^2}\right)/2$

$r^{k+1} = x^k + (t^k - 1/t^{k+1})(x^k - x^{k-1})$

end for

In Algorithm 1, $f(r^k) = \frac{1}{2}\|Rr^k - b\|^2$, $\nabla f(r^k)$ describes the gradient of the function f at point r^k, L_f and ρ are two positive constant scalars, and

$$project(x,[l,u]) = \begin{cases} x & l \le x \le u \\ l & x \le l \\ u & x \ge u \end{cases} \tag{2}$$

Pixel values of image are normalized to the range [l, u] via *project* function, where $u > l \ge 0$. Otherwise, the reconstructed image will have artifacts because of the appearance of the negative pixel values after sparsity transformation.

$$prox_\rho(g)(x) := \arg\min_u \{g(u) + (1/2\rho)\|u-x\|^2\} \tag{3}$$

Results: Fast composite splitting algorithm (FCSA) [3], contourlet based iterative thresholding method [4], and two FISTA-based methods using wavelet and curvelet regularization terms [2, 5], respectively, are utilized to compare the performance of FICOTA. The conventional four methods are simply described as FCSA, ICOTA, FIWTA and FICTA, respectively. For fair comparison, the coefficients normalizing process is also added to other methods. All algorithms are coded using Matlab 2009b on Dell PC T1500.

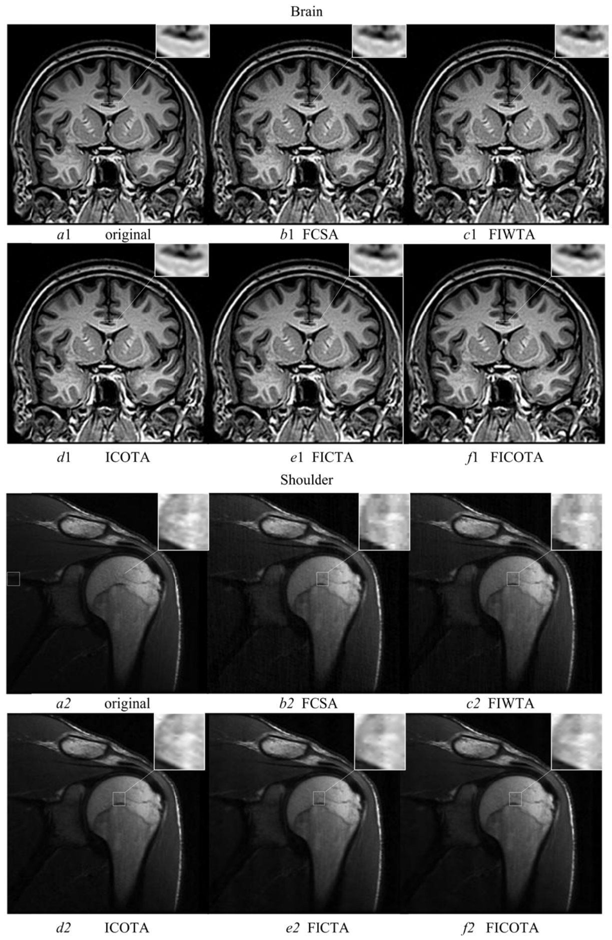

Fig . 1 *The original and reconstructed images:*
*a*1,*a*2 The original MR images.
*b*1-*f*1 Reconstructed Brain images of different algorithms.
*b*2-*f*2 Reconstructed Shoulder images of different algorithms.

Two MR images, Brain and Shoulder, are presented in Fig. 1 (a1 & a2) and used in the experiments. For convenience, the MR images were resized to the same size 256*256 and the sampling ratio is set to be approximately 20%. We used the Daubechies wavelet with two decomposition levels for FIWTA, the wrapping version of the second generation curvelet [8] for FICTA, and the redundant sharp frequency localization contourlet (SFLCT) [9] with 2^5, 2^4, 2^3, 2^2, 2^1 directional subbands from coarse to fine scales for ICOTA and FICOTA. Gaussian white noise with standard deviation 0.01 was added to the k-space measurements b. The regulation parameter α was assigned with value 0.075, and maximum iteration number of each algorithm is set as 50. The tolerance of the residue parameter of ICOTA was set as 1e-3.

To compare the performance of different algorithms, some objective criteria including signal-to-noise ratio (SNR), peak signal-to-noise ratio (PSNR) [4], transferred edge information (TEI) [4], and $L2$ norm error are also adopted.

Fig. 1 shows the reconstructed images using different algorithms. According to Fig. 1, FICOTA, ICOTA and FICTA are more effective in suppressing noise and representing edges when compared with FIWTA and FCSA.

Table 1-2 summarize the comparisons of different algorithms based on the objective criteria. The proposed FICOTA outperforms FCSA and FIWTA in term of these criteria, although its running time is slower than them. Additionally, FICTA and ICOTA can acquire comparable reconstruction quality as FICOTA, but their computation time costs are higher than the proposed FICOTA.

Table 1: Comparisons of different algorithms based on Brain.

Brain	FCSA	FIWTA	ICOTA	FICTA	FICOTA
SNR	19.80	18.62	22.10	22.16	**22.51**
PSNR	31.73	30.56	34.08	34.09	**34.44**
TEI	0.809	0.792	0.851	0.858	**0.865**
*L*2 Norm Error	0.070	0.080	0.054	0.053	**0.051**
CPU Time(s)	2.48	1.56	39.41	72.39	12.11

Table 2: Comparisons of different algorithms based on Shoulder.

Shoulder	FCSA	FIWTA	ICOTA	FICTA	FICOTA
SNR	21.58	21.47	24.45	23.51	**24.75**
PSNR	40.38	40.28	43.29	42.31	**43.55**
TEI	0.691	0.692	0.753	0.751	**0.769**
*L*2Error	0.064	0.065	0.046	0.051	**0.044**
CPU Time(s)	2.31	1.78	20.56	67.86	12.66

Conclusion: This letter proposes to combine FISTA with contourlet transform to enforce the curve sparsity of magnetic resonance images with fast computation. Experimental results show that the proposed method, FICOTA, significantly improves the quality of reconstructed images, with slightly compromised computation time compared to FISTA-based methods with wavelet regularization constraints. Compared to FICTA and ICOTA, FICOTA can reach comparable reconstruction effectiveness but use much less computation time. Future work may combine the more efficient FISTA-based methods and the more sparse transforms to improve the effectiveness and the efficiency of compressed sensing MRI.

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