Balance sparsity model for tight-frame representation in compressed sensing MRI

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***Abstract*—A balance sparsity model in compressed sensing MRI is proposed. The new model outperforms the traditional analysis or synthesis sparsity models both in vision and numerical errors when a balance parameter is set properly.**

I. INTRODUCTION

In sparse representation using tight frames, synthesis sparsity and analysis sparsity are two typical models [1], which have been used in compressed sensing MRI (CS-MRI) [3] [4] to accelerate the imaging. The balance sparsity model (BSM) balance these two models [2]. However, to our best knowledge, BSM has never been investigated in CS-MRI and its performance is still unknown. In this work, we compare these models numerically and find that balance model outperforms other two models [3][4] both in vision and numerical errors when one key parameter is tuned properly.

II. METHODS

A balance sparsity model for CS-MRI is

$$\hat{\mathbf{x}} := \boldsymbol{\psi}^*\hat{\boldsymbol{\alpha}};\ \ \hat{\boldsymbol{\alpha}} := \arg\min_{\boldsymbol{\alpha}}\left\{\frac{1}{2}\left\|\mathbf{b}-\mathbf{F}_U\boldsymbol{\psi}^*\boldsymbol{\alpha}\right\|_2^2+\lambda\left\|\boldsymbol{\alpha}\right\|_1+\frac{\beta}{2}\left\|(\mathbf{I}-\boldsymbol{\psi}\boldsymbol{\psi}^*)\boldsymbol{\alpha}\right\|_2^2\right\}$$

Where $\hat{\mathbf{x}}$ is the underlying image, $\boldsymbol{\Psi}$ is a tight frame, $\boldsymbol{\alpha}$ is the coefficient, $\mathbf{F}_U$ is k space undersampling operator, $\mathbf{b}$ is sampled data, λ is a regularization parameter and β is the balance parameter. The balance model becomes the synthesis model when $\beta=0$ and analysis model when $\beta=\infty$. Setting $0<\beta<\infty$ leads to the so called balance model [1] since it balances between analysis and synthesis models.

In this work, a proximal forward-backward splitting (PFBS) algorithm [5] speeded up by the fast iterative shrinkage thresholding (FISTA) [6] is explored to solve the balance model in CS-MRI. For comparison, the synthesis model is solved by setting β=0 and the analysis model is solved by alternating direction method of multipliers.

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III. RESULTS

The brain image (size 256×256) in Fig. 1(a) is acquired from a healthy volunteer at a 3T Siemens Trio Tim MRI scanner using the T2-weighted turbo spin echo sequence (TR/TE = 6100/99 ms, FOV=220 220 mm2, slice thickness=3 mm). The relative ℓ_2 norm error (RLNE) [4] is adopted to measure the reconstruction error. Undecimated discrete wavelet transform from Rice Wavelet Toolbox is used as the tight frame.

Reconstructed images shown in Figs. 1 (b) and (d) indicate that the analysis model produces smoother image than the synthesis model which generates sharp artifacts. With the proposed balance model, image structures (Fig. 1 (c)) are preserved best (Fig. 1(g)) and the RLNE is the smallest. In our simulation, an optimal balance parameter is set as =1.37.

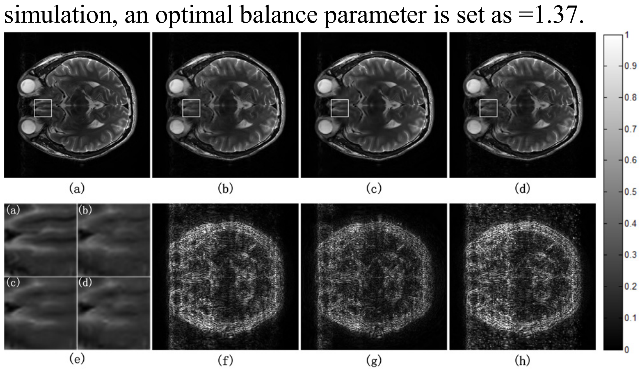

Figure 1. Reconstructed images. (a) the original image, (b)-(d) are reconstructed images of analysis, balance and synthesis models, (e) the zoom out part of (a)-(d), (f)-(g) are corresponding error images of (b)-(d). The RLNEs of (b)-(d) are 0.1143, 0.0947, 0.1221 while running time for reconstructing them are 12s, 49s, 51s. Note that 40% k-space data are sampled.

REFERENCES

[1] Elad, M., et al., Inverse problems, 2007. 23(3): p. 947.
[2] Shen, Z., International Congress of Mathematicians, 2010.
[3] Lustig, M.,et al., Magnetic Resonance in Medicine, 2007. 58(6): p. 1182-1195.
[4] Qu, X.B., et al., Magnetic Resonance Imaging, 2012. 30(7): p. 964-977.
[5] Shen, Z.W.,et al.SIAM Journal on Imaging Sciences, 2011. 4(2): p. 573-596.
[6] Beck, A. and M. Teboulle, SIAM Journal on Imaging Sciences, 2009. 2(1): p. 183-202