Appendix

Proof of equivalence between (16) and (8) in the manuscript.

Denoting that $G(\mathbf{x}) = \lambda\|\mathbf{\Psi x}\|_1 + 1/2\|\mathbf{y}-\mathbf{UFx}\|_2^2$, then one has

$$\begin{aligned} & \min_{\boldsymbol{\alpha}\in \mathrm{Range}(\mathbf{\Psi})} \lambda\|\boldsymbol{\alpha}\|_1 + \frac{1}{2}\|\mathbf{y}-\mathbf{UF\Phi}\boldsymbol{\alpha}\|_2^2 \\ \overset{(a)}{=} & \min_{\boldsymbol{\alpha}\in \mathrm{Range}(\mathbf{\Psi})} \lambda\|\mathbf{\Psi\Phi}\boldsymbol{\alpha}\|_1 + \frac{1}{2}\|\mathbf{y}-\mathbf{UF\Phi}\boldsymbol{\alpha}\|_2^2 \\ \overset{(b)}{=} & \min_{\boldsymbol{\alpha}\in \mathrm{Range}(\mathbf{\Psi})} G(\mathbf{\Phi}\boldsymbol{\alpha}) \\ \overset{(c)}{=} & \min_{\mathbf{x}\in\Omega} G(\mathbf{x}), \end{aligned} \tag{A1}$$

with $\Omega = \{\mathbf{\Phi}\boldsymbol{\alpha} \mid \boldsymbol{\alpha}\in \mathrm{Range}(\mathbf{\Psi})\}$ where (a) from the property (6) for $\boldsymbol{\alpha}\in \mathrm{Range}(\mathbf{\Psi})$, (b) and (c) are straightforward based on the definition of $G(\cdot)$ and Ω. Next, we show that $\Omega = \mathbb{C}^N$. On one hand, we have

$$\mathbf{x}\in\mathbb{C}^N \overset{\mathbf{\Phi\Psi x}=\mathbf{x}}{\Rightarrow} \mathbf{x}\in\Omega \text{ with } \boldsymbol{\alpha} = \mathbf{\Psi x}. \tag{A2}$$

On the other hand, we have

$$\begin{aligned} & \mathbf{x}\in\Omega \\ & \Rightarrow \mathbf{x} = \mathbf{\Phi}\boldsymbol{\alpha} \text{ for some } \boldsymbol{\alpha}\in \mathrm{Range}(\mathbf{\Psi}) \\ & \Rightarrow \mathbf{x} = \mathbf{\Phi}\boldsymbol{\alpha} \text{ with } \boldsymbol{\alpha} = \mathbf{\Psi}\tilde{\mathbf{x}} \text{ for some } \tilde{\mathbf{x}}\in\mathbb{C}^N \\ & \Rightarrow \mathbf{x} = \mathbf{\Phi\Psi}\tilde{\mathbf{x}} = \tilde{\mathbf{x}} \text{ for some } \tilde{\mathbf{x}}\in\mathbb{C}^N \\ & \Rightarrow \mathbf{x}\in\mathbb{C}^N \end{aligned} \tag{A3}$$

(A2) and (A3) together leads to $\Omega = \mathbb{C}^N$. This together with (A1) leads to

$$\begin{aligned} & \min_{\boldsymbol{\alpha}\in \mathrm{Range}(\mathbf{\Psi})} \lambda\|\boldsymbol{\alpha}\|_1 + \frac{1}{2}\|\mathbf{y}-\mathbf{UF\Phi}\boldsymbol{\alpha}\|_2^2 \\ = & \min_{\mathbf{x}} \lambda\|\mathbf{\Psi x}\|_1 + \frac{1}{2}\|\mathbf{y}-\mathbf{UFx}\|_2^2 \end{aligned}. \tag{A4}$$

If $\boldsymbol{\alpha}^*$ is a solution of (16) and $\mathbf{x}^*$ is a solution of (8), one has

$$G(\mathbf{\Phi}\boldsymbol{\alpha}^*) \overset{(d)}{=} G(\mathbf{x}^*) \overset{(e)}{=} G(\mathbf{\Phi\Psi x}^*) \tag{A5}$$

where (d) from the second equation in (A1) and (A4), (e) from (3). Therefore, $\mathbf{\Phi}\boldsymbol{\alpha}^*$ is also a solution of the analysis model (8) and $\mathbf{\Psi x}^*$ is also a solution of the synthesis-like model (16). This concludes the proof.