UNDERSAMPLED MRI RECONSTRUCTION WITH TRAINED DIRECTIONS FROM A GUIDE IMAGE

Xiaobo Qu[1], Di Guo[2], Bende Ning[1], Yingkun Hou[3], Shuhui Cai[1], and Zhong Chen[1]

[1]*Department of Electronic Science, Fujian Key Laboratory of Plasma and Magnetic Resonance, Xiamen University, Xiamen, Fujian, People's Republic of China*

[2]*Department of Communication Engineering, Xiamen University, Xiamen, Fujian, People's Republic of China.*

[3]*School of Information Science and Technology, Taishan University, Taian, Shandong, People's Republic of China.*

Introduction: Undersampling the k-space data can speed up magnetic resonance imaging (MRI) at the cost of introducing the aliasing artifacts. These artifacts can be obviously reduced by enforcing the sparse representation of the magnetic resonance (MR) image with respect to a pre-constructed basis or dictionary [1]. In this paper, a patch-based directional wavelets (PBDW) is proposed to sparsify the magnetic resonance (MR) image in undersampled MRI reconstruction. The geometric direction of each image patch is trained from the guide image and incorporated into the sparsifying transform to provide the sparse representation for the image to be reconstructed. Simulations demonstrate that trained PBDW leads to better edges than the convetional sparse MRI reconstruction method did.

Methods: First an image $\mathbf{x}$ is divided into patches $\mathbf{R}_j\mathbf{x}$ $(j=1,2,\cdots,J)$, then the geometric direction $\hat{\theta}_j$ of the j^{th} patch is selected out among the candidate directions $\boldsymbol{\theta}=\{\theta_1,\theta_2,\cdots\theta_d\cdots,\theta_D\}$ to minimize the approximation error

$$\hat{\theta}_j=\arg\min_{\theta_j\in\boldsymbol{\theta}}\left\|\tilde{\mathbf{c}}_j\left(\theta_j,S\right)-\boldsymbol{\Psi}^T\mathbf{P}\left(\theta_j\right)\mathbf{b}_j\right\|_2^2$$

where $\boldsymbol{\Psi}^T$ is the forward orthogonal 1D Haar wavelet, $\tilde{\mathbf{c}}_j\left(\theta_j,S\right)$ denotes the largest S-term wavelet coefficients of $\boldsymbol{\Psi}^T\mathbf{P}\left(\theta_j\right)\mathbf{b}_j$, and $\mathbf{P}\left(\theta_j\right)\mathbf{b}_j$ are the re-arranged pixels parallel to the direction θ_j [2]. A small S (S is set to be one quarter of the amounts of pixels) leads PBDW to sparsly represent the images since the lowest approximation error is found for limited wavelet coefficients. For the image $\mathbf{x}$, its representation in the PBDW is

$$\boldsymbol{\omega}=\left[\boldsymbol{\Psi}^T\mathbf{P}\left(\hat{\theta}_1\right)\mathbf{R}_1 \quad \cdots \quad \boldsymbol{\Psi}^T\mathbf{P}\left(\hat{\theta}_j\right)\mathbf{R}_j \quad \cdots \quad \boldsymbol{\Psi}^T\mathbf{P}\left(\hat{\theta}_J\right)\mathbf{R}_J\right]^T\mathbf{x}=\mathbf{A}_{\hat{\boldsymbol{\theta}}}\mathbf{x}\,.$$

Assuming the geometric directions $\hat{\boldsymbol{\theta}}=\left\{\hat{\theta}_1,\cdots,\hat{\theta}_j,\cdots,\hat{\theta}_J\right\}$ are available for all patches, MR image is reconstructed by solving

$$\hat{\mathbf{x}}=\arg\min_{\mathbf{x}}\left\|\mathbf{A}_{\hat{\boldsymbol{\theta}}}\mathbf{x}\right\|_1+\frac{\lambda}{2}\left\|\mathbf{y}-\mathbf{F}_U\mathbf{x}\right\|_2^2$$

where $\|\cdot\|_1$ stands for the ℓ_1 norm, which promotes the sparsity of all patches, and $\|\cdot\|_2$ stands for the ℓ_2 norm, which enforces the fidelity of the reconstruction to the measured k-space data. The regularization parameter λ decides the tradeoff between the sparsity and the data fidelity. Due to the lack of fully sampled k-space data, no ground truth image is avaiable to train the geometric directions. In this paper, an initial guide image is reconstructed by enforcing the sparsity of image in shift-invariant discrete wavelet (SIDWT) domain. SIDWT can mitigate blocky artifacts introduced by orthogonal discrete wavelet in conventional compressed sensing MRI methods.

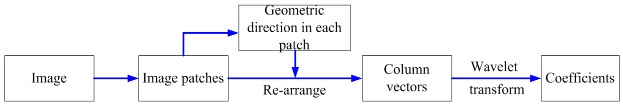

Fig. 1 Flowchart of patch-based directional wavelets

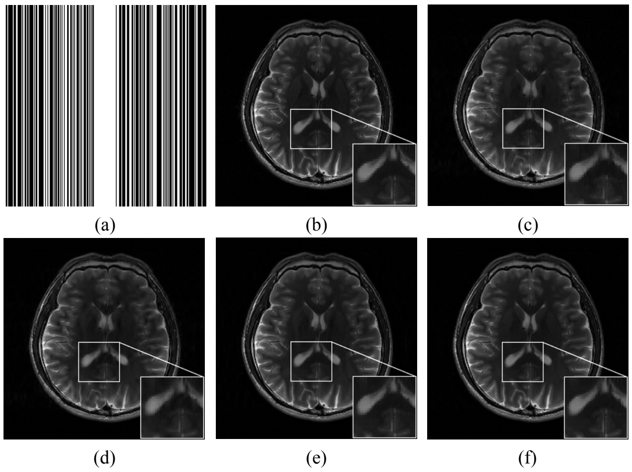

Fig.2 Reconstructed images using different methods. (a) The sampling pattern with 45% fully sampled k-space data, (b) is the fully sampled image, (c)-(d) are the reconstructed images using total variation, SIDWT, PBDW with geometric direction estimated from (e) and (b), respectively. The RLNE of (c)-(f) are 0.101, 0.107, 0.071 and 0.057.

Results: The brain image (size 256×256) in Fig. 2(b) is acquired from a healthy volunteer at a 3T Siemens Trio Tim MRI scanner using the T2-weighted turbo spin echo sequence (TR/TE = 6100/99 ms, FOV=220×220 mm^2, slice thickness=3 mm). We specify the regularization parameter $\lambda=10^6$ for total variation (TV), SIDWT and the proposed method. The relative ℓ_2 norm error (RLNE) defined as $e(\hat{\mathbf{x}})=\|\hat{\mathbf{x}}-\tilde{\mathbf{x}}\|_2 / \|\tilde{\mathbf{x}}\|_2$ is adopted to measure the difference between the reconstructed image $\hat{\mathbf{x}}$ and the fully sampled image $\tilde{\mathbf{x}}$.

The reconstructed images shown in Fig.2 indicate that the proposed method produces sharper edges (Fig.2(e)) than the conventional sparse reconstruction methods do (Figs.2(c) and (d)). The edges are further preserved with an accurate geometric information extracted from the fully sampled MR image (Fig.2(b)). At sampling rates under 0.8, the proposed method always achieves lower reconstruction error than other methods. Estimating the geometric information from SIDWT-based reconstructed image is adequate if the sampling rate is relative high (larger than 0.5 shown in Fig.3).

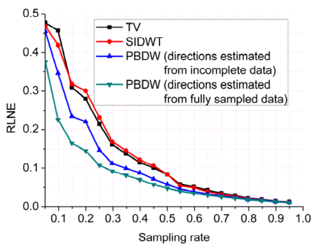

Fig.3 Reconstruction error for different sampling rates.

Conclusions: The patch-based directional wavelets (PBDW)-based sparse MRI reconstruction is proposed. Training the geometric directions from incomplete k-space data and incoporating these information into reconstruction formlation can better preserve the edges than conventional sparse reconstruction methods did.

Acknowledgement: This work was partially supported by NNSF of China (10974164 and 11174239) and the Research Fund for the Doctoral Program of Higher Education of China under Grant (200803840019). X. Qu and D. Guo were supported by Postgraduates' Oversea Study Program for Building High-Level Universities from the China Scholarship Council.

References:[1] Lustig M., *et al.*. *Magnetic Resonance in Medicine* 58 (2007) 1182-1195. [2] Peyre G. and Mallat S.. *ACM Transactions on Graphics* 24 (2005) 601-608. [3] Qu X.B. *et al.*. *Magnetic Resonance Imaging*, doi:10.1016/j.mri.2012.02.019, 2012.