全国第三届研究生数学建模竞赛

摘　　要：

参赛队号 _____

参赛密码 _____
（由组委会填写）

参赛队员姓名 蔡清波，陈云，屈小波

一、等半径区域划分模型

×

D

o

I

d_0 I

S_D D

S_I I

$[x]$ x

$o_i(x_i, y_i)$ i

R $R = 100$

$O(o_i)$ o_i R

R $R=100$

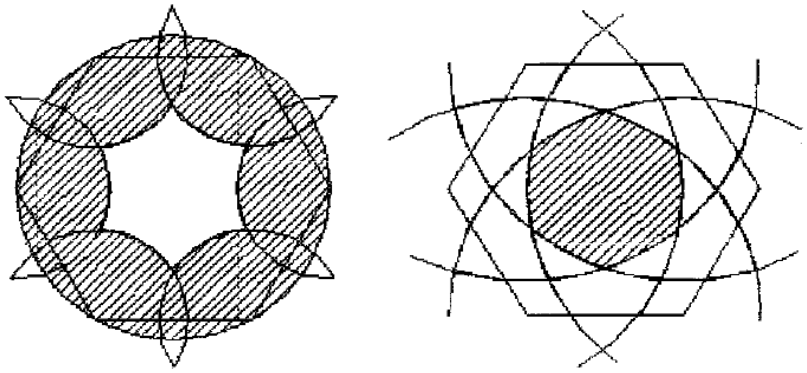
$S_{D(O(o_i),O(o_j))}$ i j

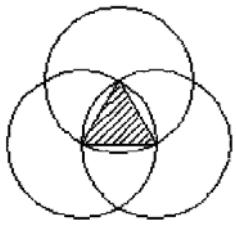
α

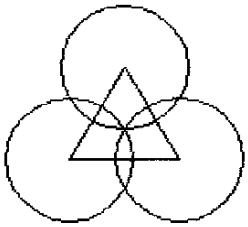
$G(o_i)$ o_i $G(o_i)=1$

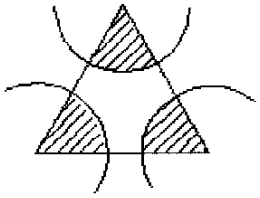
$G(o_i)=0$

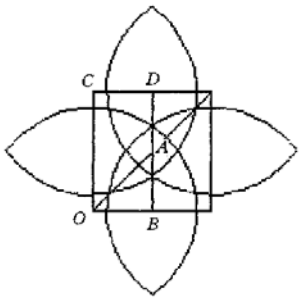
1000×1000





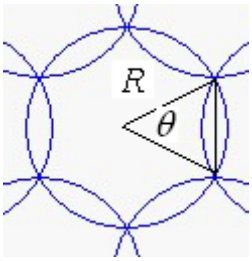






$$S_{ij} \geq \alpha\pi R^2$$

θ



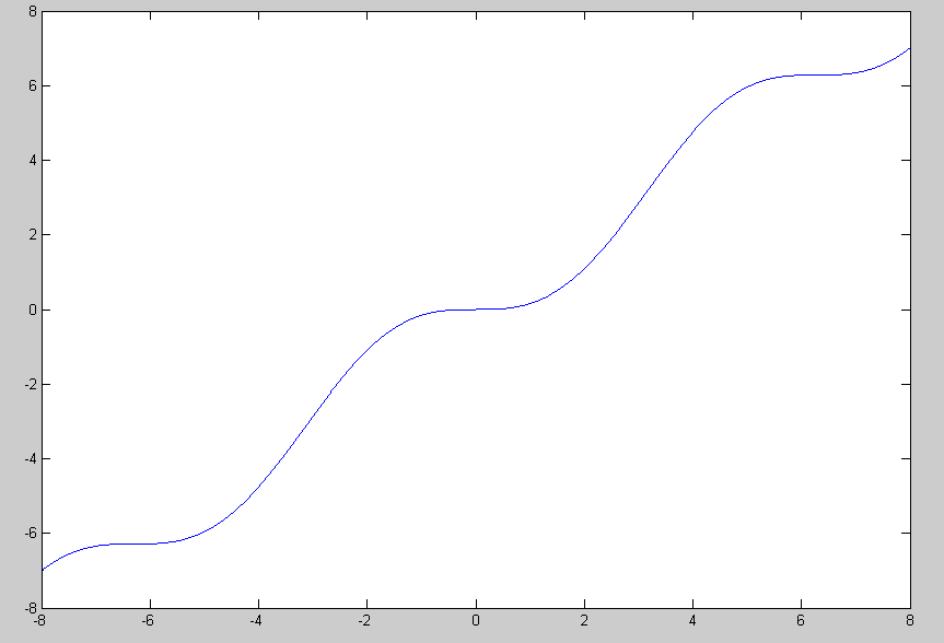
$$S_{ij} = (\pi R^2 \frac{\theta}{2\pi} - \frac{1}{2} R^2 \sin\theta) \times 2$$

$S_{ij} \geq \alpha\pi R^2$

$$\theta - \sin\theta \geq \pi\alpha$$

$\alpha = 5\%$ $\theta = 0.9968 = 57.1146^\circ$

$\alpha = 18\%$ $\theta = 1.5655$ 89.6950°



$f(\theta)=\theta-\sin\theta$ $[-8,8]$ $\theta=60^\circ$

$\alpha=0.0577>5\%$ 18% $\alpha=5\%$

$\theta=90^\circ$ $\alpha=0.1817>18\%$ $\alpha=18\%$

$$d=2R\cos\frac{\theta}{2}=2\times100\times\cos\frac{60^\circ}{2}=100\sqrt{3}$$

$$d=2R\cos\frac{\theta}{2}=2\times100\times\cos\frac{90^\circ}{2}=100\sqrt{2}$$

$\alpha\geq5\%$

$\alpha\geq18\%$

I I

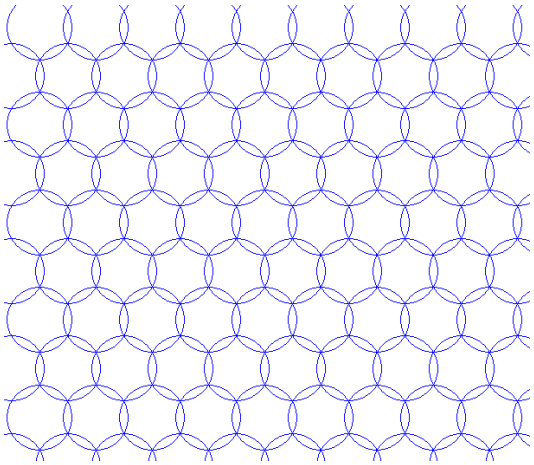
o_i $G(o_i)=1$ $G(o_i)=0$

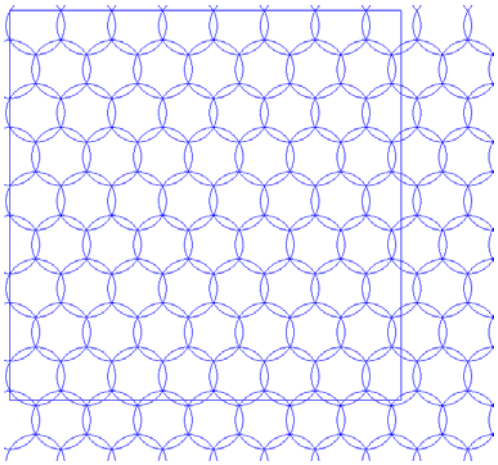
$$G(o_i)=\begin{cases}1 & O(o_i)\bigcap O(o_j)=\varnothing, O(o_i)\in I, O(o_j)\in I\\ 1 & S_{D(O(o_i),O(o_j))}\geq \alpha\pi R^2, O(o_i)\bigcap O(o_j)\neq\varnothing, O(o_i)\in I, O(o_j)\in I\\ 0 & S_{D(O(o_i),O(o_j))}<\alpha\pi R^2, O(o_i)\bigcap O(o_j)\neq\varnothing, O(o_i)\in I, O(o_j)\in I\\ 0 & O(o_i)\notin I\end{cases}$$

d_0

$$\max \quad \frac{S_{\mathrm{I}}}{\sum_i S_{O(o_i)}}$$

$$S_{O(o_i)}=G_i(o_i)\pi R^2$$





1000×1000 m I 1000×1000 m

I

$\Delta x(0\leq\Delta x<\frac{R}{2})$ $\Delta y(0\leq\Delta y<\frac{R}{2})$

$\alpha=5\%$

$\alpha = 5\%$ $d = 100\sqrt{3}$

I N_0

$$N_0 = \left[\frac{d_0}{d} \right]$$

I

$$\Delta x_{\max} = d_0 - N_0 d - kR$$

$$k = \left[\frac{d_0 - N_0 d}{R} \right]$$

$$\Delta x \in \left[0, \Delta x_{\max} \right]$$

$d = 100\sqrt{3}$ $R = 100$ $d_0 = 1000$

$$N_0 = 5$$

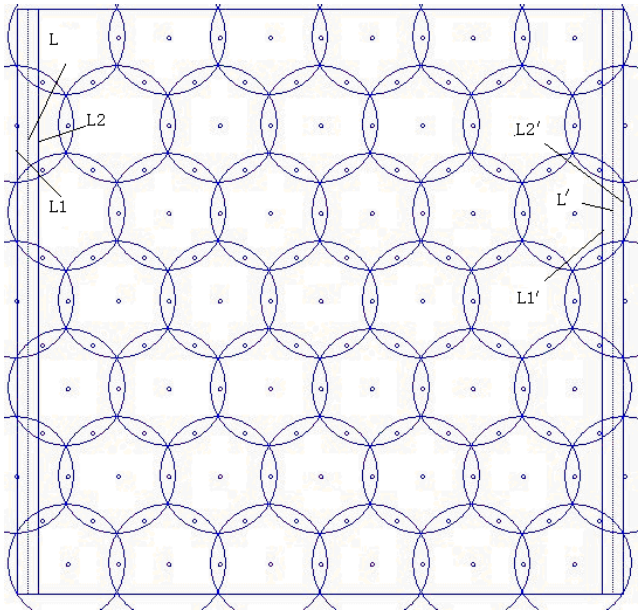
$$\Delta x_{\max} = 33.9746$$

$$\Delta x \in \left[0, 33.9746 \right]$$

$S_{O(o_i)}$ $\frac{S_{\mathrm{I}}}{\sum_i S_{O(o_i)}}$

I $l(x)$ $l_1(x_1)$

$l_2(x_2)$ $x \in [x_1, x_2]$ $l(x)$



l_1 l_1' l_2 l_2' l l' I

$x = x_1$ $x = x_2 + d_0$ l_2'

l l_1 l_1 I I

$S_{O(o_i)}$

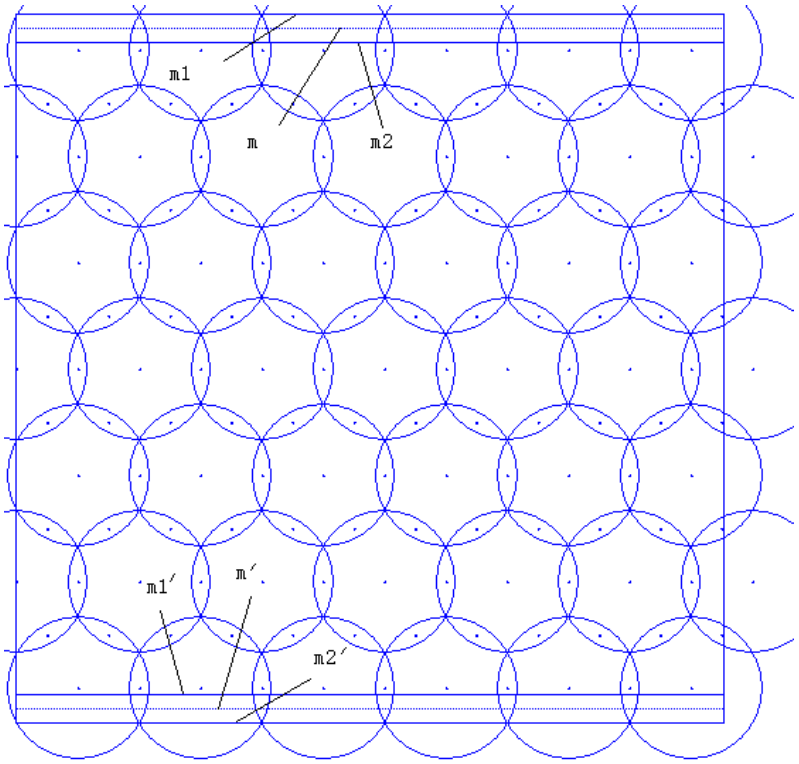
l l_2 l_2' I I $S_{O(o_i)}$

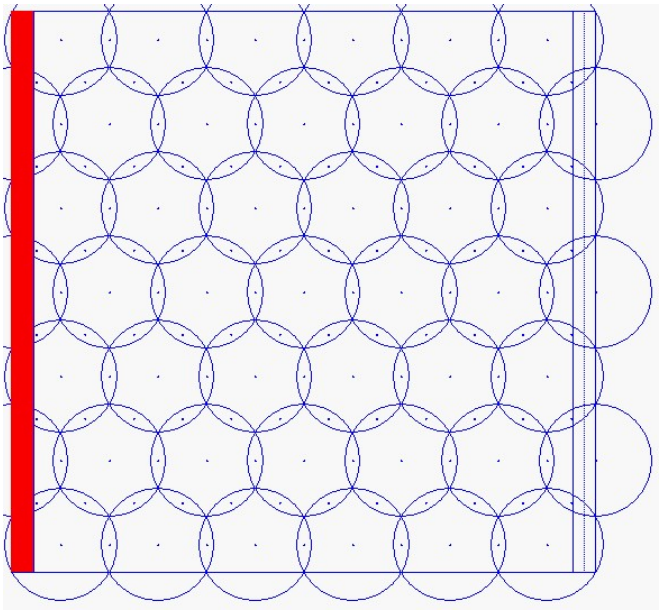
l_1 l_2 I $x \in [x_1, x_2]$ I

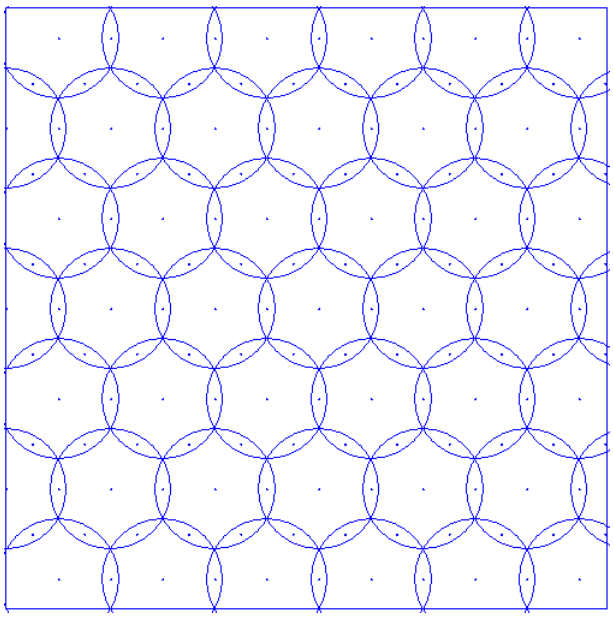
$S_{O(o_i)}$ $[x_1, x_2]$

I $m(y)$

$m_1(y_1)$ $m_2(y_2)$ $y \in [y_1, y_2]$ $m(y)$


m1
m
m2
m1′
m′
m2′





$\alpha = 18\%$

$\alpha = 18\%$ $d = 100\sqrt{2}$

I N_0

$$N_0 = \left[\frac{d_0}{d}\right] = \left[\frac{1000}{100\sqrt{2}}\right] = 7$$

I

$$\Delta x_{\max} = d_0 - N_0 d - kR = 1000 - 7 \times 100\sqrt{2} - 0 \times 100 = 10.0505$$

$$\Delta x \in [0,10.0505]$$

$$\Delta y \in [0,10.0505]$$

$x = x_1$ $x = x_2 + d_0$ l_2' l_1 l_2

I $x \in [x_1, x_2]$ I $S_{O(o_i)}$

$[x_1, x_2]$ $y = y_1$

$y = y_2 + d_0$ m_2' I

$y \in [y_1, y_2]$ I $S_{O(o_i)}$

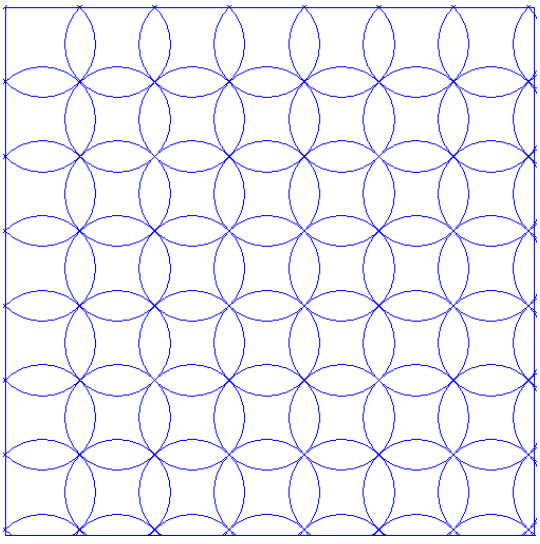
$[y_1, y_2]$

$D(x, y), x \in [x_1, x_2], y \in [y_1, y_2]$ $\alpha = 18\%$ $\dfrac{S_{\mathrm{I}}}{\sum_i S_{O(o_i)}}$

$$\left.\frac{S_{\mathrm{I}}}{\sum_i S_{O(o_i)}}\right|_{\max} = 201.0619$$

I

$$\left.\sum_i G(o_i)\right|_{\min} = 64$$



$$\alpha = 5\%$$

$$\frac{10}{45}\times\frac{3}{4}+\frac{35}{45}=94.44\%$$

$\alpha = 18\%$

$$\frac{4}{64}+\frac{24}{64}+\frac{36}{64}\times\frac{4}{5}=88.75\%$$

二、带椭圆的区域划分模型

D

S_D D

$T(D)$ D

D

$T_{\min}(D)$ D

I

d_0 I

E

$\overline{E}$ I E $\overline{E} \bigcap E = \varnothing, \overline{E} \bigcup E = I$

$r_i(x_i, y_i)$ i $75 \le r_i \le 100$

$o_i(x_i, y_i)$ i

$O(o_i, r_i)$ o_i r_i

R

$\widetilde{R}$

N

$\widetilde{N}$

$S_{D(O(o_i),O(o_j))}$ i j

$\widetilde{O}(\tilde{o}_i, \widetilde{R})$ $\tilde{o}_i$ $\widetilde{R}$

$U\left(o_i, r_i, D\right)$ $O(o_i, r_i)$ D

$J\left(o_i, r_i, D_1, D_2\right)$

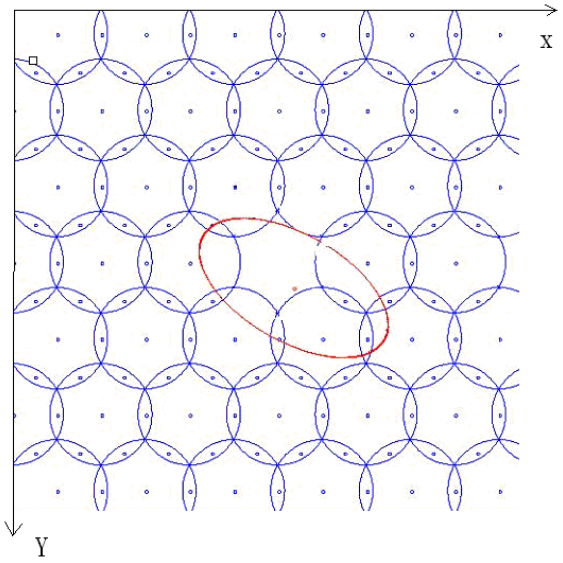
$$J\left(o_i, r_i, D_1, D_2\right) = \begin{cases} 1 & U\left(o_i, r_i, D_1\right) \le U\left(o_i, r_i, D_2\right) \\ 0 & U\left(o_i, r_i, D_1\right) > U\left(o_i, r_i, D_2\right) \end{cases}$$

$O(o_i, r_i)$ D_1 D_2 $J = 1$

$O(o_i, r_i)$ D_1 D_2 $J = 0$

I

I



$R_i = 75$ $R_i = 100$ $\alpha = 5\%$

$R_i = 75$

I

$$\sum_i G(o_i)\Big|_{\min} = 85$$

$$R_i = 75 \qquad T_{\min} = \sum_i R_i G(o_i)\Big|_{\min} = 85 \times 75 = 6375$$

$$R_i = 100 \qquad T_{\min} = \sum_i R_i G(o_i)\Big|_{\min} = 45 \times 100 = 4500$$

$R_i \in [75,100]$

I N_0 $\Delta x \in [0, \Delta x_{\max}]$

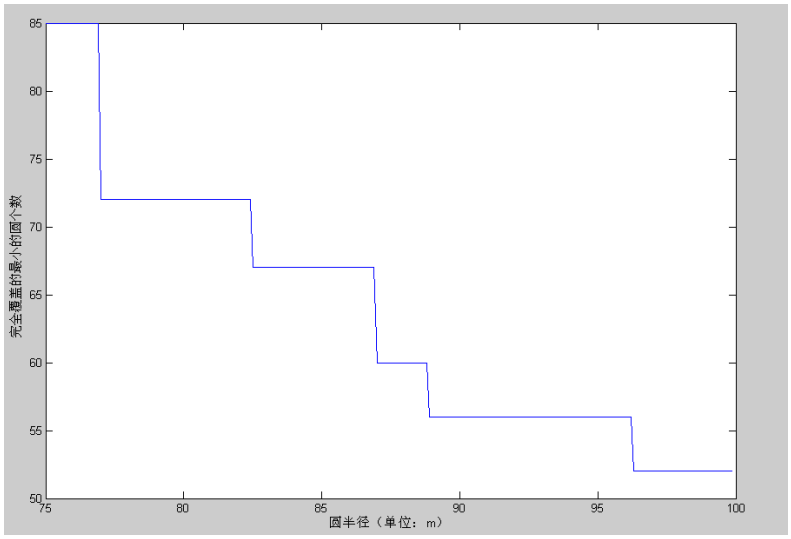
$\Delta y \in [0, \Delta y_{\max}]$ $\Delta r = 0.1$

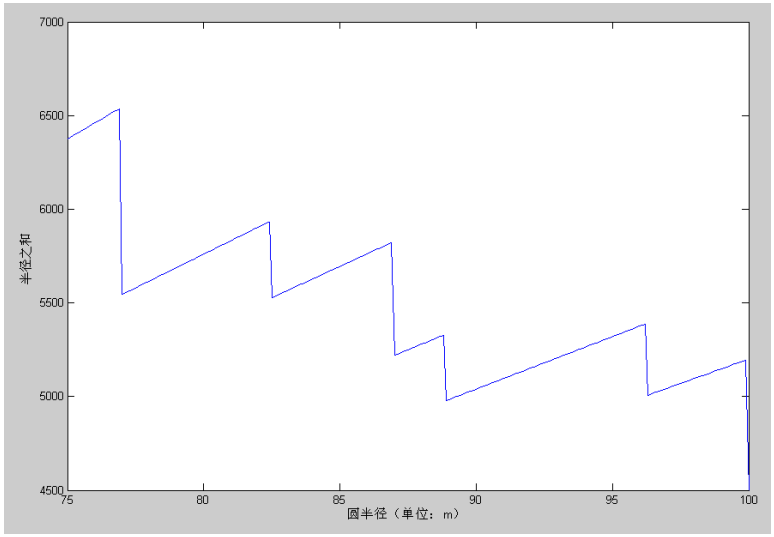
$R_i = 75 + i \times 0.1 \times k \quad (0 \le k \le 250, k \in Z)$

I

I $k = 250, R_i = 100$

T





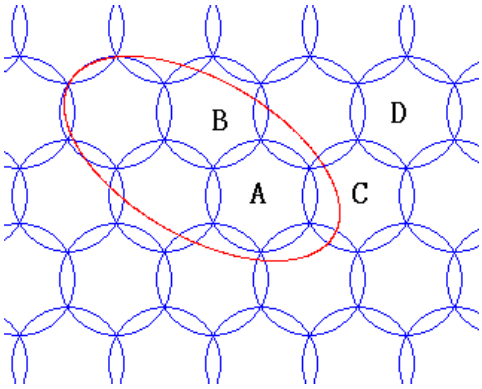
I

$R_i = 90, R_i = 95 \quad \sum_i G(o_i)\Big|_{\min} = 56$ I

$\sum_i G(o_i)\Big|_{\min}$ T

I

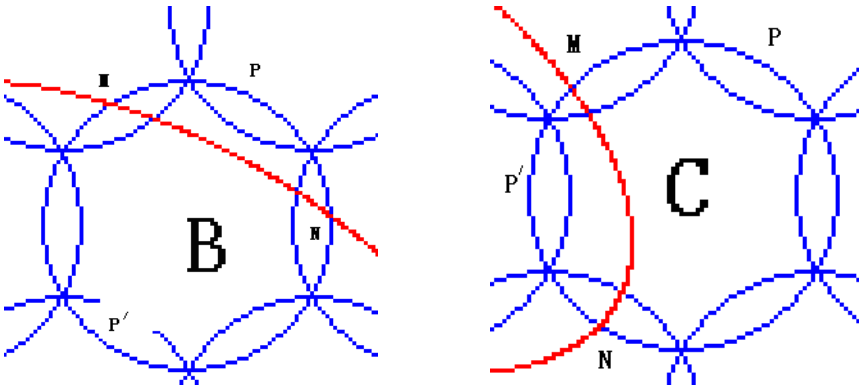
I



$\overline{E}$ $\widehat{MPN}$ E

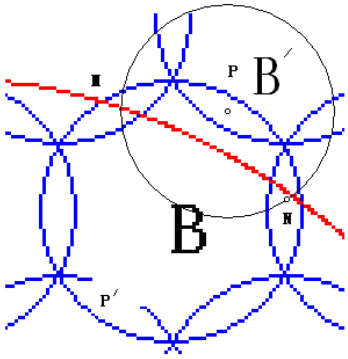
$\widehat{MP'N}$ $0 < \dfrac{\widehat{MPN}}{\widehat{MP'N}} \le 1$

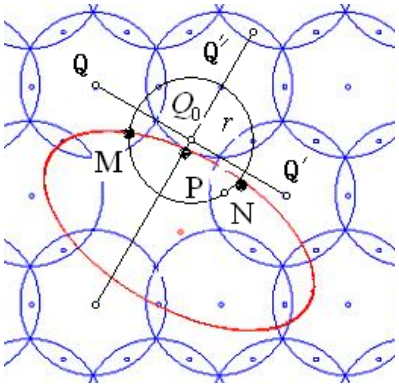
$\overline{E}$ $\widehat{MPN}$ E $\widehat{MP'N}$ $\dfrac{\widehat{MPN}}{\widehat{MP'N}} > 1$



$\overline{E}$

B′





Q Q' Q'' $\overset{\frown}{MNP}$

Q_0 Q'' QQ' QQ'

Q_0 QQ' r Q_0

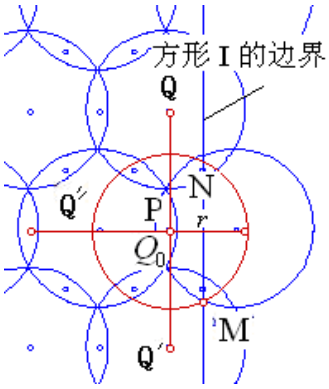
Q Q' Q'' α 5% $\alpha = 5\%$

I

Q_0 QQ' r Q_0

Q Q' Q'' α 5%

$\alpha = 5\%$



I

I

$\widehat{MPN}$ $\widehat{M'PN'}$

r $\widehat{M'PN'}$

T

I

$\widetilde{R}$

$\widetilde{N}$ E r $\overline{E}$

$\sum_i G(o_i) = \widetilde{N}$ I

$\overline{E}$ $\overline{E}$

$$\min \quad T(\overline{E}) = \widetilde{N}\widetilde{R} + \sum_i J\left(o_i(x_i, y_i), r_i, E, \overline{E}\right)\Delta r(o_i(x_i, y_i), \Delta x, \Delta y)$$

$$\begin{cases} \widetilde{N} = \sum_i G(o_i) \\ 75 \le r_i \le 100 \\ 75 \le r_i + \Delta r \le 100 \\ 0 \le \Delta x \le \Delta x_{\max} \\ 0 \le \Delta y \le \Delta y_{\max} \end{cases}$$

$$\Delta r(o_i(x_i, y_i), \Delta x, \Delta y) = G(o_i(x_i + \Delta x, y_i + \Delta y)\left(r_i(x_i + \Delta x, y_i + \Delta y) - \widetilde{R}\right)$$

$$\Delta x_{\max} = d_0 - N_0 d - \left[\frac{d_0 - N_0 d}{R}\right] R$$

$$\Delta y_{\max} = d_0 - N_0 d - \left[\frac{d_0 - N_0 d}{R}\right] R$$

$$J\left(o_i(x_i, y_i), r_i, E, \overline{E}\right) = \begin{cases} 1 & U\left(o_i, r_i, E\right) \le U\left(o_i, r_i, \overline{E}\right) \\ 0 & U\left(o_i, r_i, E\right) > U\left(o_i, r_i, \overline{E}\right) \end{cases}$$

$$G(o_i(x, y)) = \begin{cases} 1 & O(o_i) \bigcap O(o_j) = \varnothing, O(o_i) \in \overline{E}, O(o_j) \in \overline{E} \\ 1 & S_{D(O(o_i), O(o_j))} \ge \alpha \pi R^2, O(o_i) \bigcap O(o_j) \ne \varnothing, O(o_i) \in \overline{E}, O(o_j) \in \overline{E} \\ 0 & S_{D(O(o_i), O(o_j))} < \alpha \pi R^2, O(o_i) \bigcap O(o_j) \ne \varnothing, O(o_i) \in \overline{E}, O(o_j) \in \overline{E} \\ 0 & O(o_i) \notin \overline{E} \end{cases}$$

$\alpha \ge 5\%$ $\Delta r = 0.1$

$$T_{\min}\left(I\right) = \sum_i G(o_i)\widetilde{R}\Big|_{\min} = 4500$$

$$\widetilde{N} = \frac{T_{\min}\left(I\right)}{\widetilde{R}} = 45$$

$$\widetilde{R} = 75$$

E I

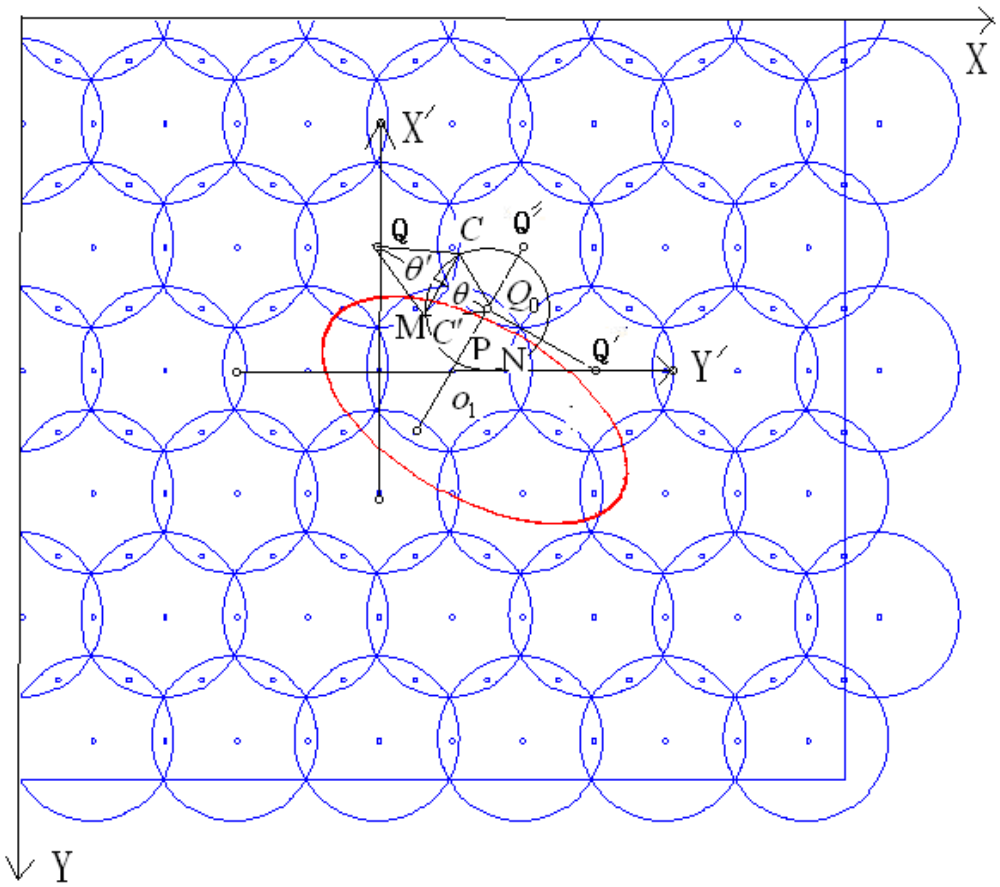
$y = e\left(x\right)$ I i $\left(x - x_i\right)^2 + \left(y - y_i\right)^2 = \widetilde{R}^2$

$$\begin{cases} y = e(x) \\ (x - x_i)^2 + (y - y_i)^2 = \widetilde{R}^2 \end{cases}$$

i $M_i\left(\widetilde{x}_i, \widetilde{y}_i\right)$ $N_i\left(\widetilde{x}_i', \widetilde{y}_i'\right)$

$o_i\left(x_i, y_i\right)$ $\widehat{M_i P_i N_i}$ $\widehat{M_i P_i' N_i}$ $\widehat{M_i P_i N_i} \in \overline{E}, \widehat{M_i P_i' N_i} \in E$

$\widetilde{O}(o_1, \widetilde{R})$ $\widetilde{O}(o_2, \widetilde{R})$ $\widetilde{O}(o_2, \widetilde{R})$



$\widetilde{O}(o_1, \widetilde{R})$ $\widehat{M_i P_i N_i}$

$O(Q_0\left(x_0, y_0\right), r)$ $Q_0\left(x_0, y_0\right)$ $Q\left(x_Q, y_Q\right)$

$Q'\left(x_{Q'}, y_{Q'}\right)$

$$\begin{cases} x_0 = \dfrac{x_Q + x_{Q'}}{2} \\ y_0 = \dfrac{y_Q + y_{Q'}}{2} \end{cases}$$

$S_{D(O(Q),O(Q_0))}$ $O(Q)$ $O(Q_0)$ $S_{D(O(Q'),O(Q_0))}$

$O(Q')$ $O(Q_0)$ $S_{D(O(Q''),O(Q_0))}$ $O(Q'')$ $O(Q_0)$

α 5%

$$\begin{cases} S_{D(O(Q),O(Q_0))} \geq \alpha\pi\widetilde{R}^2 = 5\% \times 100^2\pi \\ S_{D(O(Q'),O(Q_0))} \geq \alpha\pi\widetilde{R}^2 = 5\% \times 100^2\pi \\ S_{D(O(Q''),O(Q_0))} \geq \alpha\pi\widetilde{R}^2 = 5\% \times 100^2\pi \end{cases}$$

$S_{D(O(Q),O(Q_0))}$ $O(Q)$ $O(Q_0)$ $C(x_c, y_c)$ $C'(x_{c'}, y_{c'})$

$|CC'|$ Q θ' Q_0 θ

$$\theta' = \arccos(\frac{|CC'|^2 - 2\widetilde{R}^2}{2\widetilde{R}^2})$$

$$\theta = \arccos(\frac{|CC'|^2 - 2r^2}{2r^2})$$

$S_{D(O(Q),O(Q_0))}$ $\dfrac{\theta'\widetilde{R}^2}{2} - -\widetilde{R}^2 \sin\theta'$

$S_{D(O(Q),O(Q_0))}$ $\dfrac{\theta'\widetilde{R}^2}{2} - -\widetilde{R}^2 \sin\theta' + \dfrac{\theta r^2}{2} - -r^2 \sin\theta$

$$\frac{\theta'\widetilde{R}^2}{2} - -\widetilde{R}^2 \sin\theta' + \frac{\theta r^2}{2} - -r^2 \sin\theta \geq \alpha\pi\widetilde{R}^2$$

$O(Q_0)$ $O(o_1)$

$$r \geq |MQ_0|$$

$|MQ_0| = \sqrt{(x_M - x_Q)^2 + (y_M - y_Q)^2}$

r

$$\begin{cases} 75 \le r \le 100 \\ \dfrac{\theta' \widetilde{R}^2}{2} - - \widetilde{R}^2 \sin\theta' + \dfrac{\theta r^2}{2} - - r^2 \sin\theta \ge \alpha\pi\widetilde{R}^2 \\ r \ge \sqrt{\left(x_M - x_Q\right)^2 + \left(y_M - y_Q\right)^2} \end{cases}$$

Q_0 QQ' $O(Q)$ $O(Q')$ Q_0Q''

$$S_{D(O(Q'),O(Q_0))} \ge \alpha\pi\widetilde{R}^2 = 5\% \times 100^2\pi$$

$S_{D(O(Q''),O(Q_0))} \ge \alpha\pi\widetilde{R}^2 = 5\% \times 100^2\pi$ $O(Q_0)$ $O(Q'')$

$T_{\min}\left(\overline{E}\right)$ $T(\overline{E})\Big|_{\min}$ 4402.1

$\overline{E}$

E

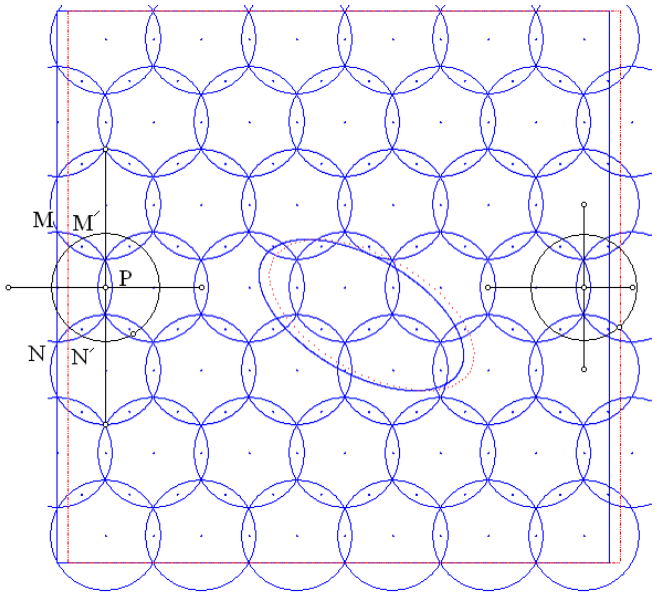
$\overline{E}$ E

I $T(\overline{E})$ Δx Δy

$\Delta x \in [0, \Delta x_{\max}]$ $\Delta y \in [0, \Delta y_{\max}]$

5%

r $T(\overline{E})$



I

$\widehat{MPN}$ $\widehat{M'PN'}$

Δx Δy r

Δx Δy r Δx Δy r

$\Delta r = 0.1$ $r_i = 75 + i \times 0.1 \times k$ $(0 \le k \le 250, k \in Z)$

$$\frac{0.1}{100} \times 100\% = 0.1\%$$

$\Delta x \in [0, \Delta x_{\max}]$ $\Delta y \in [0, \Delta y_{\max}]$ $r \in [75,100]$

$T(\overline{E})$

I

$$T_{\min}(\overline{E}) = T(\overline{E})\big|_{\min} = 4327.1$$

$\overline{E}$

$\overline{E}$

E

$\overline{E}$ E

T r $\overline{E}$

$\sum_i G(o_i)$ I

$\overline{E}$ $\overline{E}$

三、一跳覆盖区模型

a_{ij} i j $a_{ij}=0$

$a_{ij}=1$

A

$N_{total}(i)$

$V=\{1,2,...,N\}$ N

$S(\overline{E})$ $\overline{E}$

N N

$$A=\left[a_{ij}\right],\ \ \mathrm{a_{ij}}\in\{o,1\},\ \ \mathrm{i,j}=1,2,...,N$$

$a_{ij}=0$

$a_{ji}=1$ $a_{ji}=1$ N

A

$a_{ji}=0$ $a_{ji}=1$

$V=\{1,2,...,N\}$ N $N_{adj}(i)$

$N_{total}(i)$

$N_{total}(i)=N_{adj}(i)\bigcup\{i\}$ C_i

$N_{total}(i)$

C_i C_i

$N-N_{total}(i)$ C_j

$N_{total}(j)\subset 3N-N_{total}(i)$ C_j

$$\min \quad S(\overline{E})=\sum_i G(o_i)r_i(o_i)$$

$$\begin{cases} 75 \le r_i(o_i) \le 100 \\ G(o_i) = \begin{cases} 1 & O(o_i) \bigcap O(o_j) = \varnothing, O(o_i) \in \overline{E}, O(o_j) \in \overline{E} \\ 1 & S_{D(O(o_i),O(o_j))} \ge \alpha\pi R^2, O(o_i) \bigcap O(o_j) \ne \varnothing, O(o_i) \in \overline{E}, O(o_j) \in \overline{E} \\ 0 & S_{D(O(o_i),O(o_j))} < \alpha\pi R^2, O(o_i) \bigcap O(o_j) \ne \varnothing, O(o_i) \in \overline{E}, O(o_j) \in \overline{E} \\ 0 & O(o_i) \notin \overline{E} \end{cases} \end{cases}$$

$DN(i)$

$$DN(i) = (\sum_{j=1}^{N} a_{ij}) - 1$$

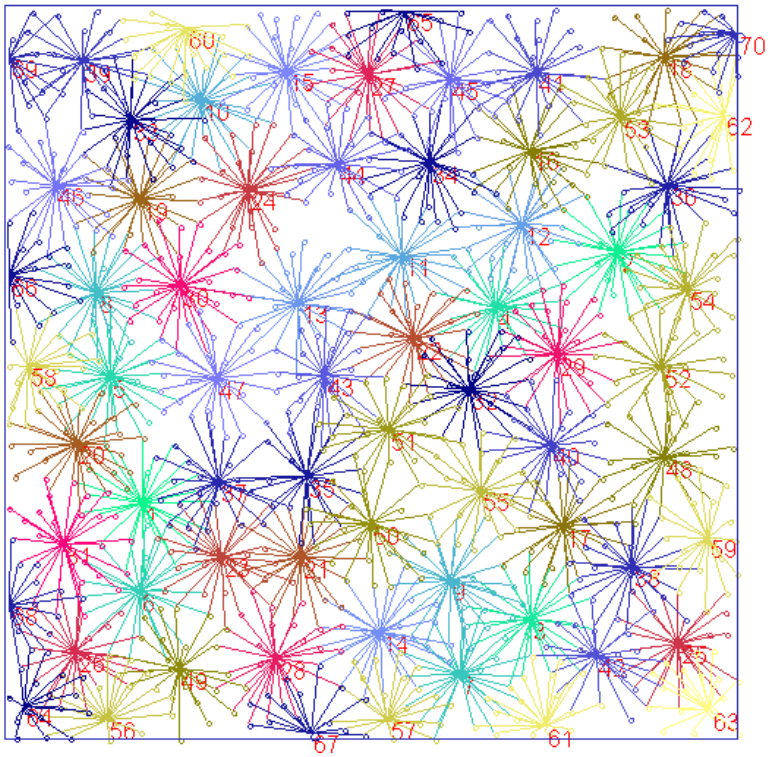
$i = 1, 2, ..., N$ $k(k \in V)$ $DN(k)$

$$DN(k) = \max_{i \in V}[DN(i)]$$

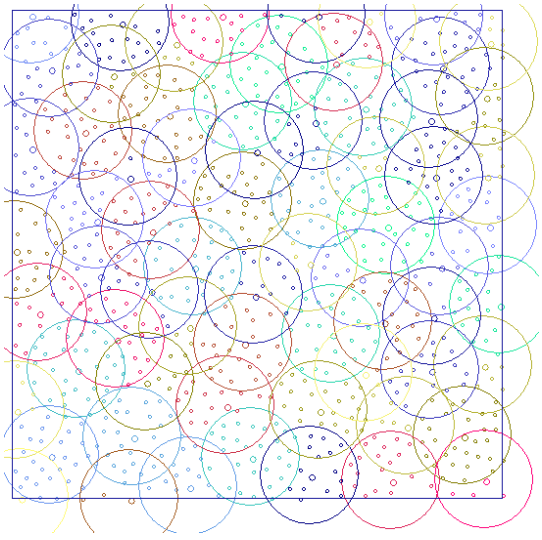
H_1 $k' \in V - N_{total}(i)$ $i' \in V - N_{total}(i)$

$$DN(k') = \max_{i' \in V - N_{total}(i')}[DN(i')]$$

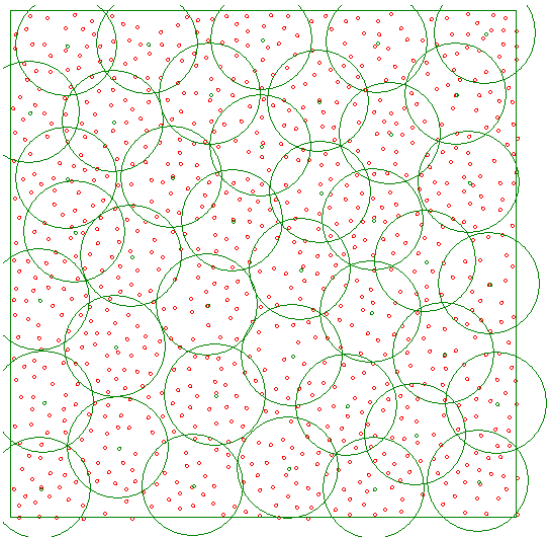
H_2



3.1



3.2



3.3